Compatibility of Neutron Star Masses and Hyperon Coupling Constants*

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It is shown that the modern equations of state for neutron star matter based on microscopic calculations of symmetric and asymmetric nuclear matter are compatible with the lower bound on the maximum neutron-star mass for a certain range of hyperon coupling constants, which are constrained by the binding energies of hyperons in symmetric nuclear matter. The hyperons are included by means of the relativistic Hartree– or Hartree–Fock approximation. The obtained couplings are also in satisfactory agreement with hypernuclei data in the relativistic Hartree scheme. Within the relativistic Hartree–Fock approximation, hypernuclei have not been investigated so far.

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Neutron star matter (NSM), bound by gravity, differs from high density nuclear matter produced in heavy ion reactions in several respects: 1) Since the repulsive Coulomb force is much stronger than the gravitational attraction, NSM is much more asymmetric than terrestrial matter in heavy ion collisions. 2) The weak interaction time scale is small in comparison with the lifetime of the neutron star (NS), but large in comparison with the lifetime of high-density matter in heavy ion reactions. For that reasons dense matter in high-energy reactions has to obey the constraints of isospin symmetry and strangeness conservation whereas NSM is a charge neutral system with no strangeness conservation (see, for instance, [1] with further references). Extraterrestrial NSM is for that reasons a rather theoretical object with a very complex structure. Since its density stretches over an enormous range, reaching from crystalline iron on the surface to several times of nuclear matter saturation density in the core, one encounters in the theoretical descriptions many obstacles, for which one has to introduce theoretical assumptions and extrapolations [1, 2]. However one can try to combine both high density systems in a common theory, which is possible in modern field theoretical approaches. The first systematic investigation in this respect was performed by Glendenning [3], who used the standard nuclear relativistic Lagrangian extended by inclusion of hyperons and deltas, i. e.

$$\mathcal{L}(x) = \sum_{B=p,n,\Sigma^{\pm\,o},\Lambda,\Xi^{o\cdot-\cdot},\Delta^{-\cdot o\cdot+\cdot++}} \mathcal{L}_B^0(x) + \sum_{M=\sigma,\omega,\pi,\rho,\delta} \left\{ \mathcal{L}_M^0(x) + \sum_{B=p,n,\ldots,\Delta} \mathcal{L}_{B,M}^{\mathrm{int}}(x) \right\} + \sum_{L=e^-,\mu^-} \mathcal{L}_L(x) , \quad (1)$$

where the interaction is mediated by the mesons $\sigma, \omega, \cdots$ and the leptons are treated as free particles. Furthermore, a scalar meson self-interaction is included. More explicitly, the hadron part is given by

$$\mathcal{L}_H(x) = \sum_B \bar{\psi}_B(x) \left[i\gamma^\mu \partial_\mu - m_B + g_\sigma^B \sigma(x) - g_\omega^B \gamma^\mu \omega_\mu(x) - f_\omega^B \frac{\sigma^{\mu\nu}}{4m_B} F_{\mu\nu}^\omega - \frac{f\pi}{m\pi} \gamma^5 \gamma^\mu \boldsymbol{\tau}_B \cdot \partial_\mu \boldsymbol{\pi} \right.$$

$$\left. - \frac{1}{2} g_\rho^B \gamma^\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(x) - f_\rho \frac{\sigma^{\mu\nu}}{4m_B} \boldsymbol{\tau} \cdot \boldsymbol{F}_{\mu\nu}^\rho \right] \psi_B(x) + \frac{1}{2} \left[\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma^2 \sigma^2(x) \right]$$

$$+ \frac{1}{2} \left[\partial_\mu \boldsymbol{\pi}(x) \cdot \partial^\mu \boldsymbol{\pi}(x) - m_\pi^2 \boldsymbol{\pi}^2(x) \right] - \frac{1}{4} \boldsymbol{F}_{\mu\nu}^\rho(x) \cdot \boldsymbol{F}^{\mu\nu,\rho}(x) + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu(x) \cdot \boldsymbol{\rho}_\mu(x) - \frac{1}{4} F_{\mu\nu}^\omega(x) F^{\mu\nu,\omega}(x)$$

$$+ \frac{1}{2} m_\omega^2 \omega^\mu(x) \omega_\mu(x) - \frac{1}{3} m_N b_N [g_\sigma \sigma(x)]^3$$

$$- \frac{1}{4} c_N [g_\sigma \sigma(x)]^4$$

with

$$F^\omega_{\mu\nu}(x) \equiv \partial_\mu\omega_\nu(x) - \partial_\nu\omega_\mu(x) \,,$$
$$\boldsymbol{F}^\rho_{\mu\nu}(x) \equiv \partial_\mu\boldsymbol{\rho}_\nu(x) - \partial_\nu\boldsymbol{\rho}_\nu(x) \,. \tag{3}$$

Within this scheme, the vast majority of investigations was performed using coupling parameters in the nucleonic sector adjusted to the nuclear matter parameters (ρ_o, E/A, K, J), the Dirac mass $\tilde{m}$ and a variety of choices for the hyperon couplings (see, for instance, [1, 3 - 5]. The calculations for such Lagrangians could be performed without numerical convergence problems and resulted in a high strangeness contents of NSM [1, 3]. However it was pointed out in two more recent investigations, that due to the selected high Dirac mass ($\simeq 0.79\ m_N$) such parametrizations are neither in accordance with standard phenomenological parametrizations [6], as, for instance, the frequently employed parametrizations NL1 and NL-SH, or with parametrizations adjusted to the outcome of microscopic relativistic Brueckner–Hartree–Fock calculations (RBHF), respectively, which give a lower Dirac mass of approximately 0.6 m_N [7, 8] (necessary also, for instance, for a correct reproduction of the spin-orbit-splitting [6]). Attempts to use directly such parametrizations in NSM–calculations, which one should prefer theoretically, have not been successful, since they lead to negative nucleon Dirac masses by inclusion of hyperons [1, 7]. This strong decrease of $\tilde{m}$, caused by the stronger σ–field in the case of lower Dirac masses at saturation in comparison to the high choice of $\tilde{m}$ by Glendenning, seems to be also not in accordance with the behaviour of $\tilde{m}$ in RBHF–calculations for higher densities in symmetric nuclear matter [9]. For that reason one has to correct this feature of the RH, which occurs also in the relativistic Hartree–Fock–approximation, by a slight change of the Lagrangian for higher densities, which gives a more moderate decrease in this region, similiar to the RBHF–values but does not change the equation of state (EOS) in the region up to moderate densities above saturation nuclear density [1]. Within this scheme, described in detail in [1], where the changes in the masses are moderate and the high–density behaviour is through the dominance of the ω–repulsion hardly altered, one obtains now in the RH–approximation NS–matter compositions with a high strangeness component which are qualitatively similar [1] to former investigations, but have now

the advantage that they are based on a correct phenomenological description of nuclear properties and the outcome of microscopic RBHF–calculations, with an additional assumption, done in a controlled and changeable manner by one additional parameter. A closer inspection reveals that all the other previous attempts, where such problems did not arise at a first glance, have made more or less implicitly, for instance, by selecting a high Dirac mass [3, 5] or by introducing an ω self-interaction [10], such an additional assumption, however with a rather specific enforcement of the high density behaviour on the Dirac mass (for details, see [1]). In the RH–scheme, Δ's usually do not occur, since due to the large ρ–coupling, needed to reproduce the correct symmetry coefficients, the charge-favoured but isospin-unfavoured Δ^- is suppressed [1, 3]. Since the data from the hypernuclei are not yet conclusive with respect to the hyperon couplings (for more details, see [11, 12]), it is common in more modern investigations to use the SU(6) symmetry for the vector couplings and adjust the σ–coupling with respect to the lowest hyperon level in nuclear matter. However, the data from hypernuclei still permit a large bandwidth for the couplings [1, 6, 11 - 13]. As it was pointed out by Glendenning and Moszkowski [13], one can also use the maximum neutron-star mass of at least 1.5 solar mass, the so-called Oppenheimer–Volkoff limit M_{OV}, as an additional constraint besides the binding energy of the hyperon in matter, to fix the permitted pairs of coupling constants $x_\sigma = g_{\sigma H}/g_{\sigma N}(< 1)$ and $x_\omega = g_{\omega H}/g_{\omega N}(\leq 1)$. However one has to keep in mind that such fixations depend strongly on the chosen nucleonic parametrization in combination with the selected approximation [1]. It seems therefore worthwhile to revisit this problem in a short note in view of the discussed theoretically more satisfying parametrizations, based on the outcome of RHBH–calculations with the modern Brockmann–Machleidt OBE-potentials [1, 7, 14]. The parametrizations are given in Tables 1 and 2.

More subtle is the case of the RHF–approximation [4, 14], where the constraint of the hyperon binding energy in nuclear matter is difficult to implement [1]. A further new feature is that due to the exchange contributions the g_ρ–coupling becomes smaller in the direction of the values of the realistic OBE-potentials, with the net effect that now the Δ's can play an important role in the composition of NSM, so decreasing the strangeness contents [1]. The Δ–component
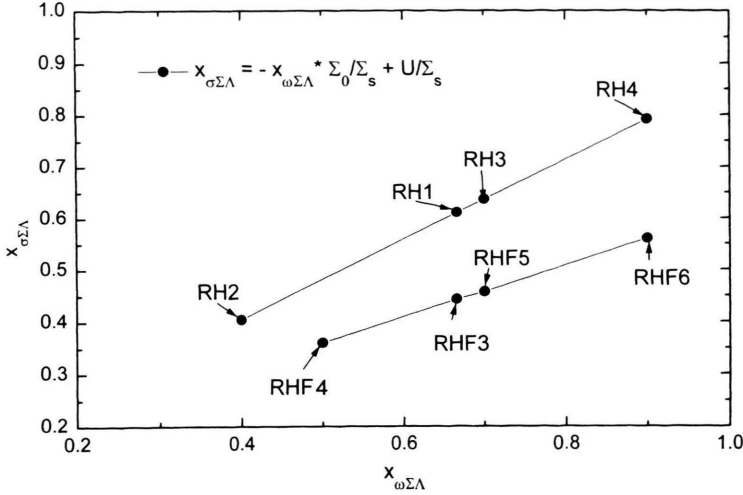
Fig. 1. Constraint of the relative σ–coupling constant for a given ω–coupling due to the lowest hyperon binding energy in saturated nuclear matter for the RH– and RHF–approximation, respectively.

Table 1. Parametrizations of the RH– and RHF–Lagrangian adjusted to the RBHF–calculations. For the masses the following values were selected (MeV): $m_N = 939$, $m_\sigma = 550$, $m_\omega = 738$, $m_\pi = 138$, $m_\rho = 770$ ($g_\pi = 1.00265$, $f_\pi^2/4\pi = 0.08$). The parametrizations are labelled as follows: RHA $\hat{=}$ RBHFA etc. for the potential A (effective mass at the Fermi surface $\tilde{m} = 617.8$ MeV (A); 621.8 MeV (B)).

	g_σ	g_ω	g_ρ	$10^3 \times b_N$	$10^3 \times c_N$	f_ρ/g_ρ
RHA	9.58096	10.67698	3.81003	3.333665	-3.52365	–
RHF A1	9.28353	8.37378	2.10082	3.333689	–2.15239	–
RHF A2	9.24268	8.25548	2.19809	2.96514	–2.68614	3.7
RHF A3	9.16665	8.07540	2.37987	1.95524	–2.36335	6.6
RHB	9.59169	10.68084	3.66541	3.62616	–4.17140	–
RHF B1	9.36839	8.40466	1.77326	3.74354	–3.18456	–
RHF B2	9.33266	8.32154	1.86078	3.44306	–3.46261	3.7
RHF B3	9.26782	8.19391	2.02216	2.67292	–3.18198	6.6

Table 2. Relative coupling strengths of the hyperons in the relativistic Hartree approximation. In [1] occured a misprint for $x_{\sigma\Lambda}(= 0.676)$ and $x_{\sigma\Xi}(= 0.342)$. For RH1 – RH4, $x_{\sigma H}$ was adjusted for a selected $x_{\omega H}$ according to the Van–Hove–Hugenholtz theorem, which gives for the hyperon potential depth $U(H) = x_{\sigma H}\Sigma_s + x_{\omega H}\Sigma_o$, where Σ_s and Σ_o are the scalar and vector part of the self-energy at saturation. For comparison we included also the universal coupling (RH5) and the free Fermi gas approximation for the hyperons (RH6). For the nucleonic sector the parametrization RHB was used.

Approximation	$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$x_{\sigma\Xi}$	$x_{\omega\Xi}$	$x_{\sigma\Delta}$	$x_{\omega\Delta}$
RH1(SU(6))	0.611	2/3	0.346	1/3	1	1
RH2	0.40	0.40	0.40	0.40	1	1
RH3	0.64	0.7	0.63	0.7	1	1
RH4	0.79	0.9	0.79	0.9	1	1
RH5	1	1	1	1	1	1
RH6	0	0	0	0	0	0

Table 3. Relative coupling strengths of the hyperons in the relativistic Hartree–Fock approximation. In RHF1 and RHF2 the coupling strengths of the RH–approximation were used. For RHF3 – RHF11, x_σ was adjusted for a given x_ω according to the binding energy of the hyperon in symmetric nuclear matter at saturation. For the nucleonic sector, the parametrization RHF B1 was used. RHF10 is characterized by fixing the potential depth of Δ^- to the same value as in RH1 (for the composition, see [1]).

Approximation	$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$x_{\sigma\Xi}$	$x_{\omega\Xi}$	$x_{\sigma\Delta}$	$x_{\omega\Delta}$
RHF1	0.611	2/3	0.346	1/3	1	1
RHF2	0.79	0.9	0.79	0.9	1	1
RHF3	0.44	2/3	0.26	1/3	1	1
RHF4	0.36	0.5	0.35	0.50	1	1
RHF5	0.46	0.7	0.45	0.7	1	1
RHF6	0.56	0.9	0.55	0.9	1	1
RHF7	1	1	1	1	1	1
RHF8	0.44	2/3	0.26	1/3	0.625	0.625
RHF9	0.44	2/3	0.26	1/3	0.625	1
RHF10	0.44	2/3	0.26	1/3	0.75	1
RHF11	0.44	2/3	0.26	1/3	0	0

is rather sensitive to the choice of the Δ–coupling, and one obtains qualitatively rather different compositions, reaching from the standard picture for lower Δ–couplings, as recommended by Rapp et al. [15], to Δ–dominated compositions with much smaller strangeness obtained for $g_\Delta/g_N = 1$ [1]. Since a final

solid statement about the choice of these couplings cannot be made, one cannot exclude, at present, the possibility that the composition of NSM might deviate
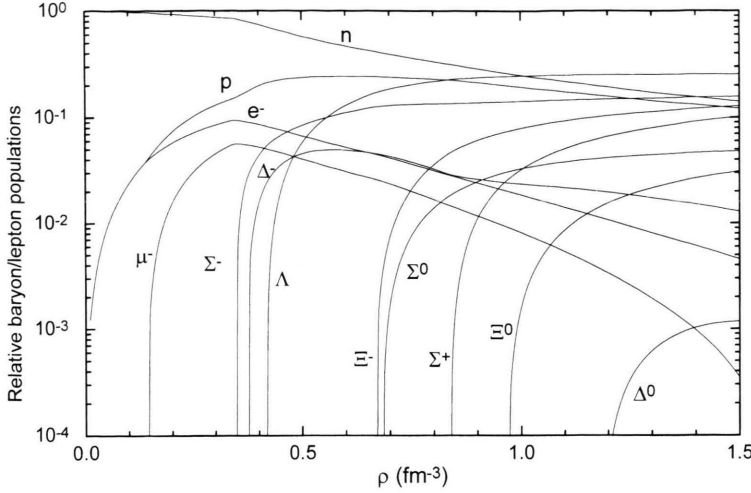
Fig. 2. Baryon-lepton composition as function of the density of an NS in the RH–approximation with the strongest hyperon–coupling (RH4).
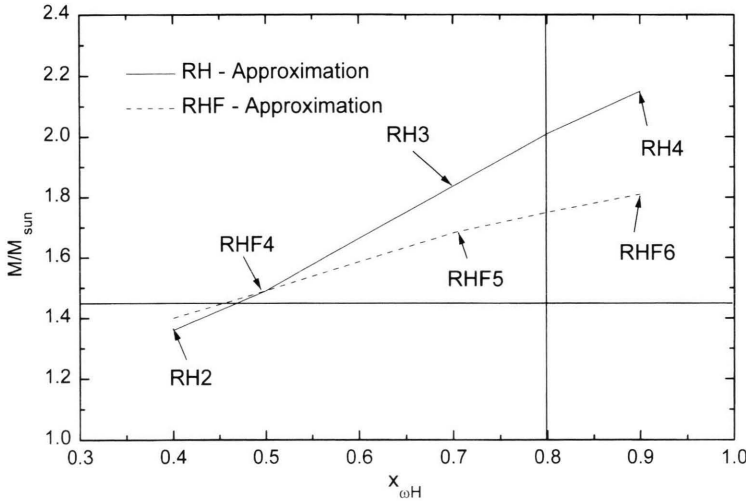

Fig. 3. Oppenheimer–Volkoff masses for static neutron stars for the different approximations as function of the relative $x_{\omega H}$–coupling. $x_{\sigma H}$ is adjusted for these parametrizations according to the lowest hyperon binding energy in matter. The horizontal line gives the mass of the pulsar PSR 1913+16. $x_{\omega H} = 0.8\,(x_{\sigma H} < 0.72)$ gives the highest value for the coupling constant compatible with properties of hypernuclei [12, 20].

from the standard picture of a high strangeness component, which may be a special feature of the RH–approximation with a large ρ–coupling (for details, see [1]). The chosen parametrizations are given in Table 3. The dependence of the relative scalar coupling for the hyperons for a selected relative vector coupling is exhibited in Figure 1. As far as the RH–approximation is concerned, all parametrizations with $x_\omega \geq 0.5$ give maximum masses above 1.6 $M_\odot$, the lowest maximum gravitational masses are obtained for no coupling, i.e. RH6, where one obtains $M_{OV} = 1.17\,M_{OV}$. The Oppenheimer–Volkoff masses for the SU(6)–values are: $M_{OV} = 1.68\,M_\odot$ with a baryonic mass of $M_{OVB} = 1.92\,M_\odot$ for nonrotating NSs, and $M_{OV} = 2.03\,M_\odot (M_{OVB} = 2.32\,M_\odot)$ for NSs rotating with their Kepler frequency $\Omega_K =$

$7.97 \times 10^3\mathrm{s}^{-1}$. The OV–masses increase with stronger hyperon-coupling to approximately 2.15 $M_\odot(\Omega = 0)$ or 2.45 $M_\odot(\Omega = \Omega_K)$, respectively, for the EOS–RH4. In this case, with the strongest hyperon interaction one obtains the largest hyperon contribution, which is shown in Figure 2. Since the masses of most of the pulsars are in the range of 1.5 $M_\odot$ (the most accurately measured mass is that of the PSR 1913+16 with $M/M_\odot = 1.422 \pm 0.003$ [16], all pairs of hyperon couplings seem to be compatible or at least not in contradiction with the observations. The comparison for the RH is depicted in Figure 3. In the framework of the RHF–approximation [1, 4, 14] the situation is more complicated. Naive use of the hyperon couplings from the RH–treatment is not compatible with the binding energies of hyperons in saturated

nuclear matter, since in this case the exchange contributions contribute strongly different in the time-like and scalar parts of the self-energies and hence cause an overestimation of the Λ–binding energy in matter. In this case, the choice with SU(6) vector couplings is not sufficient to reach an OV–mass of 1.5 $M_\odot$ (M_{OV} = 1.35 $M_\odot$). By correcting this feature, one obtains lower scalar couplings (see Table 3) in comparison with the RH. The OV–masses corresponding to the SU(6) Λ–coupling, i.e. RHF3, give now M_{OV} = 1.59 $M_\odot$ (M_{OVB} = 1.86 $M_\odot$) for a static, nonrotating spherical NS, and M_{OV} = 1.88 (M_{OVB} = 2.20 $M_\odot$) for the deformed NS rotating at the Kepler frequency $\Omega_K = 10.51 \times 10^3 \text{s}^{-1}$. For this EOS the Δs contribute significantly, since the strong coupling of the Δs in combination with the weaker ρ–coupling is capable to overcome their iso-spin unfavoured deficit. However decrease of the Δ–coupling according to [15] is sufficient to push out again the iso-spin unfavoured Δs and the hyperons contribute in a similiar manner as in the RH. The results for the OV–masses (RHF9) are then in the same range, namely: M_{OV} = 1.65 $M_\odot$ (M_{OVB} = 1.89 $M_\odot$) for the static NS and M_{OV} = 1.94 $M_\odot$ (M_{OVB} = 2.22 $M_\odot$) for the rotating star with the Kepler frequency $\Omega_K = 8.26 \times 10^3 \text{s}^{-1}$. The outcome for the RHF is depicted in Fig. 3, too. From Figs. 1 and 3 one can infer that the parametrizations with $0.5 \leq x_{\omega\Lambda} \leq 0.8$ for the RH– and RHF–approximations are with respect to the hyperon couplings in accordance with the NS–mass data, since they are capable to sustain a mass above 1.5 $M_\odot$, which is in accordance with the observation of the pulsar PSR 1913-16 with a mass of 1.44 $M_\odot$. In this context one should remark that for an isolated NS the maximum mass is smaller than the OV–mass emerging from the calculation of a cold NS. This difference is caused by the fact that NSs at birth are composed of so-called supernova matter with a high lepton fraction characterized by neutrino trapping, an almost constant entropy per baryon, and a high temperature comparable to the Fermi energies of the particles involved. As a result, the EOS is stiffer than for cold NSM, with the consequence that the maximum mass of the hot NS is reached for smaller baryon numbers. Therefore, despite the fact that the maximum masses for cold and hot matter hardly differ, one obtains for an isolated NS (no accretion) a smaller constraint on the

maximum mass, since the baryon number must be equal for both stars. The difference depends on the chosen EOS and the specific assumptions in the evolution of the hot NS, but a fair estimate gives an upper limit of 0.7 $M_\odot$ (see for instance [16, 17]), so that this reduction is still in accordance with a mass of 1.44 $M_\odot$. As far as the comparison with relativistic calculations of hypernuclei is concerned, one has restricted oneself to the use of the RH and has fixed furthermore the vector couplings to the SU(6) values and adjusted $x_{\sigma\Lambda}$ to the binding energy of a Λ in the s-state of $^{17}_\Lambda$O. One has obtained, for instance in [18], $x_{\sigma\Lambda}$ = 0.621, $x_{\sigma\Sigma}$ = 0.621 and $x_{\sigma\Xi}$ = 0.375, which agrees rather well with RH1, if one takes into consideration that the nucleonic sector was parametrized differently, namely via using NL-SH [9]. But also the stronger couplings may be possible according to the fits of Schaffner et al. [12]. If one uses an RH–approach with density-dependent couplings adjusted to the outcome of our RBHF–calculations, one needs a slightly lower $x_{\sigma\Lambda}$ = 0.59 in order to obtain the correct lowest binding energy of Λ in ^{17}O, [20]. In view of the included density-dependence such deviations seem reasonable [20]. For the RHF–approximation a calculation of Λ–hypernuclei has not been performed so far.

In conclusion, we have determined a range of possible EOSs for NSM, based on microscopic EOSs of symmetric and asymmetric nuclear matter, not possessing the unjustified high Dirac masses of earlier treatments, and which are mutually compatible with neutron-star masses, Λ–binding in nuclear matter and hypernuclear levels (in the RH). The relation between the scalar and vector coupling was fixed by the binding energy, for instance, of the lowest Λ–level in nuclear matter by the Hugenholtz–Van Hove theorem. So far the comparison has been only tested for the relativistic Hartree–approximation, since hypernuclei calculations in the RHF–approximation are not available at present.

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